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Mathematics News Letter

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To mathematics in general, to the following causes in particular is this journal dedicated: (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M. A. of A. and N. C. of T. of M. projects.

VOL. 4 BATON ROUGE, LA., DECEMBER, 1929 No. 4

BY-PRODUCTS OF MATHEMATICAL STUDY

In a final analysis it will be found that the finest and most worthwhile things in life are its by-products. Pleasure, recreation, the social programs, even art, music, literature, are, for the great majority of us, collateral rather than primary objects of effort. The consuming interest with which a botanist searches for a rare specimen of plant life and the keen thrill of delight felt by him when he finds it are by-products only, yet, but for this emotional element in his research, his research would not go far.

Similarly, while the disciplines of study are largely but by-products of continued mental application directed to some specific practical objective, in numberless, if not in the majority of cases, the disciplines are of far greater value than the objective itself.

The **habit** of keeping one's ideas in organized form is more apt to be developed from proper mathematical study than it is from study in any other field. **Habituation** of the mind to the practice of system in the working out of any definite task is

more apt to result from sufficiently prolonged study of mathematics than from study in nearly an other field for the same length of time. The **habit** of concentrating successfully for a long period of time upon any subject requiring a considerable measure of thought is more likely to be acquired from mathematical study than from any other study. **Maximum increase** of the **power** to **generalize** is a more probable consequence of mathematical study than of study in other fields.

All such by-products however, with many more that could be mentioned, must necessarily be dependent upon a proper study of mathematics, and that is **not a proper** study of mathematics which does not make habitual use of well-ordered, well-organized, concepts, habitual use of systematic method, habitual concentration, habitual generalization.

—*S. T. S.*

INSPIRING!

University of Washington
Department of Mathematics
Seattle

January 10, 1930.

Professor S. T. Sanders,
Louisiana State University,
Baton Rouge, La.

Dear Sir:

I dont know to whom I am indebted for the copy of the April (1929) Mathematics News Letter which reached me the other day, but I was glad to receive it and shall place an order for it for our mathematics reading room. Let me compliment you on the excellence, both as to contents and the appearance, of the Letter. I think it should prove an invaluable aid to the high school teachers and be not without interest to most college teachers of mathematics.

Your note on the Factorization of the Third Order Continuant caught my eye, and the result is the note which I enclose herewith, and which I hope you may find suitable for a place in the

News Letter. If not, give it an honorable burial in your waste basket.

If you print it, please send me 25 copies of the issue containing it for circulation in the Pacific Northwest.

Yours very sincerely,

ROBERT E. MORITZ.

Comment on the above is needless. Professor Moritz is a widely known scholar, and of course the editors are pleased to furnish a place for the note referred to. It appears in the present issue. What must be equally gratifying to us all are his appreciative remarks about the News Letter. —*S. T. S.*

SHOULD WE GO TO CLEVELAND, MISSISSIPPI?

The joint meeting of college and high school mathematics teachers is instructive, entertaining and inspiring. To mingle with one's friends of the mathematical fraternity is a privilege always enjoyed by every one. You come away mathematically refreshed. You come away thoroughly saturated with the idea that mathematics is the foundation stone of the sciences.

Your enthusiam lasts for months and is shown in your class room work. Your pupils, your principal, your school, and your town feel the effect of your visit to this meeting. You carry back, along with this spark of mathematical inspiration, some real knowledge of mathematical work that will prove helpful. Why not attend this meeting and make the teaching of mathematics a privilege—not a job? —*H. S.*

AN INVITATION REPEATED

Competent and experienced mathematical workers are invited to furnish contributions to the Mathematics News Letter on such topics as the following:

Mathematics and the principle of election in a liberal arts college program.

The mathematical mind and the non-mathematical mind—is the difference real?

Methods of motivating interest in mathematics.

Sketches of successful mathematicians.

Mathematics and school administrations.

Personality in mathematics teaching.

Review of important articles in the Mathematics Teacher.

Review of important articles in the American Mathematical Monthly.

News notes from college and high school mathematics departments of Louisiana and Mississippi.

—The Editors.

AN APPEAL FOR THE NEWS LETTER REPEATED

The adoption and successful operation of a scheme of cooperation between the M. A. of A. on the one hand and the Council on the other, has attracted the attention of national leaders in the mathematical field. Again we make appeal for a more united financial support of the News Letter, on the part of Louisiana and Mississippi.

There are, approximately, one thousand high school and college mathematics teachers in Louisiana and Mississippi. IT IS THROUGH THE AGENCY OF SUCH A LOCAL ORGAN AS THE NEWS LETTER THAT EVERY HIGH SCHOOL MATHEMATICS DEPARTMENT IS ABLE TO RENDER OR TO RECEIVE A SERVICE ONCE A MONTH TO OR FROM SOME OTHER DEPARTMENT, COLLEGE OR HIGH SCHOOL, IN THE TWO STATES.

No more powerful argument for the News Letter can be advanced than that its pages reflect and promote the joint and correlate interests of secondary and college mathematics and mathematics teaching. It is the circulating organ of a union of the two great classes of mathematical workers, such union having been effected in order to bring about a MASS ADVANCE of all forms of mathematical activity in this part of the South. Among these are to be placed: Increase of professional interest on the part of mathematical workers, a growing sense of the importance of contacts between a mathematics teacher and his fellow workers, increased valuation of scholarship as a factor in successful teaching, a sharpened vision of the importance of cooperation between high school and college mathematical workers.

Properly used, the Mathematics News Letter can be made the means of bringing to our teachers of mathematics in the home territory the most recent tested-out advances in teaching projects and in mathematical course materials.

—The Editors.

COOPERATION

By HENRY SCHROEDER
Ruston High School, Ruston, La.

Is there need for cooperation on the part of the high school teachers of mathematics in order to prepare students for college? The function of the high school is not merely to supply the raw material for colleges. Since such a large per cent of high school graduates go to college, the high school teacher should be concerned about preparing students for the successful pursuit of college courses. No course in college depends more upon the foundation laid in high school than mathematics. Are the high school teachers inquiring and searching to find out if their work is successful along these lines? Or are the teachers merely proceeding along without making any attempt to find out about results?

Teaching in a town where a college is located, I have paid some attention to examinations given the freshman classes as well as to the report of the college professor concerning the mathematical foundation of the freshman. From these reports and examinations one is convinced that there is much room for improvement and much need for cooperation. The reports give some insight as to what the college demands, and wherein the high school graduate is deficient. The Department of Pure Mathematics of the University of Texas decided last fall to give each freshman section an examination in his high school work. Each student was instructed to record on his paper the name of the high school from which he received his training in mathematics. Each instructor was responsible for the grading of the papers from his sections. These grades were then turned over to a member of the department and were filed and classified according to high school.

The results were as follows:

	1928-29	1929-30
Number of schools from which students came	355	
Number of students taking examination	880	900
Number of students failing examination	571	566
Number of students passing examination	309	334
Percent of students passing examination	33.9	37.1

Of course very few of these students came from Louisiana and Mississippi. But what would be the results if statistics were taken from universities and colleges within these states? This is a matter that should concern every high school teacher, and convince every one of them that there is need for coooperation.

That the examination given by the University of Texas was not difficult, but one that every high school graduate should be able to pass, the readers of the News Letter are given an opportunity to judge for themselves.

The following is the set of questions submitted:

(1) $1/2+3/7-2/5=?$

(2) Reduce 5/13 to a decimal (3 places)

(3) Factor completely x^4-16x^8

(4) Solve the equations:

(a) $ax/c+D=Bx+a$

(b) $3x^2-17x+10=0$

(5) The width of a room is 3/4 of its length. If the width was 4 ft. more and the length 4 ft. less, the room would be square. Find its dimensions.

(6) In the triangle ABC, DE is parallel to AC, and AD=10, DB=4, BE=3. Find BC.

(7) Simplify: $$\frac{\dfrac{a}{a-b}-\dfrac{b}{a+b}}{\dfrac{b}{a-b}-\dfrac{a}{a+b}}$$

(8) Rationalize the denominator of $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

INEQUALITY PRINCIPLES

By S. T. SANDERS
Louisiana State University

Kronecker wanted to reduce all of mathematics to the language of positive and negative integers. Inequality, if we may judge from the amount of it existing in nature, has, apparently as much of the divine stamp on it as has the integer. Was it Kronecker, or another, who said that all numbers except the integers, are made by man, but integers are God made?

The uses of inequality relations obtain widely in mathematical processes, particularly in the branches which undertake to define and to apply the irrational and the transcendent numbers. They are prominent, indeed, essential, in the theory of limits.

In the following we undertake to set up what one may call the alphabet of inequality theory.

(1). When the Elements a, b of an Inequality Are Positive Real Numbers.

Definition. The expression $a<b$ means "a is less than b", or, in equivalent language, "b is greater than a", in which a and b are the inequality elements

(a) If $a<b$ then $ka<kb$, k real and positive,
$a^n<b^n$, n real, positive, or fractional.
(b) If $a<b$ and $c<d$, $ac<bd$, c,d positive, real.
(c) If $a<b$ and $c<d$, $a+c<b+d$, c,d real, positive.
(d) If $a<b$ and $b<c$, $a<c$.

Corollary. Element c is positive, since, by (1), a and b are positive.

Corollary. Inequality laws (1)(a), (1)(b), (1)(c), (1)(d), appertaining to positive elements and positive multipliers only, become equality laws if the symbol "$<$" is replaced by the symbol "$=$".

Definition. If the inequality "$a<b$" is operated on with numbers n_1 and n_2 with the result that $n_1a>n_2b$ the order of inequality is said "not to hold", or the inequality symbol is "reversed."

(2). When the Elements $-a$, $-b$ of an Inequality Are Negative Real Numbers.

(a) If $-a<-b$ then $a>b>o$.

(b) If $-a<-b$ $-ka<-kb$, k positive, real.
$ka>kb$

(c) If $-a<-b<-c$, $-a<-c$

(d) If $-a<-b$ then $(-a)^n<(-b)^n$ if n is an odd integer.

$(-a)^n>(-b)^n$ if n is an even integer.

(e) If $-a<-b$ and $-c<-d$ then $-a-c<-b-d$, $-d$, $-c$ being negative real numbers.

(f) If $-a<-b$ and $-c<-d$, $-c$, $-d$ being negative, real, then $ac>bd$.

(g) If the corresponding elements of an even number of inequalities are multiplied the symbol of the resulting inequality will be reversed if the elements are negative, but if the number of such inequalities is odd the inequality symbol will be unchanged.

(h) If $-a<-b$, $(-a)^{m/n}<(-b)^{m/n}$ holds for n, m, positive, odd.

(3) When One Element Is Positive and the Other Negative.

(a) If $-a<b$ then $-ka<kb$, k, a, b positive, real.

(b) If an odd number of inequalities of type (3)(a) are multiplied together the inequality symbol will be unchanged.

(c) If $-a<b$, then $(-a)^{1/n}<(b)^{1/n}$, n positive, odd, whole.

(d) If $-a<b$ and $c<d$, a, b, c, d real, positive, then $-ac<bd$ and $-a+c<b+d$

(4) An inequality operation which does not fall under one of the above types will in some cases result in an unchanged inequality symbol while in others it will be changed. Put otherwise, if we agree not to reverse the inequality symbol whatever the resulting inequality shall be, the order of the latter will in some cases be true and in others false.

Examples. 1. While $2<3$ and $4<7$, subtracting corresponding elements, we have the false inequality $-2<-4$.

2. On the other hand, while $2<3$ and $5<5.5$ if we subtract

CONCERNING THE FACTORIZATION OF A CERTAIN SYMMETRIC DETERMINANT OF ORDER n

By PROFESSOR R. F. MORITZ
University of Washington

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corresponding elements, the true inequality results, namely, $-3<-2.5$.

3. If we multiply corresponding elements of the inequalities $-4<3$ and $-5<10$ the true inequality, namely, $20<30$, results.

4. On the other hand if we treat similarly inequalities $-4<-3$ and $-5<2$ we get the false inequality, namely, $20<-6$.

CONCERNING THE FACTORIZATION OF A CERTAIN SYMMETRIC DETERMINANT OF ORDER *n*

In the April number of the Mathematics News Letter Professor Sanders shows that

$$\begin{vmatrix} a, b, c \\ c, a, b \\ b, c, a \end{vmatrix} = a^3+b^3+c^3-3abc=(a+b+c)(a+bw+cw^2)(a+bw^2+cw),$$

where w is one of the imaginary cube-roots of unity. He furthermore raised the question as to the existence of a corresponding formula, involving the imaginary nth roots of unity, in the case of the analogous determinant of the nth order.

The purpose of this paper is to answer the question raised by Professor Sanders.

Consider the determinant of order n

$$\begin{vmatrix} a, b,c, \dots, m,n \\ n, a,b, \dots, l, m \\ m,n,a, \dots, k, l \\ \dots\dots\dots\dots \\ b, c,d, \dots, n, a \end{vmatrix} .$$

in which the rows, or columns, of elements are the cyclic permutations of the elements in the first row, or column.

Let w represent any primitive (special) nth root of unity, then the first n successive powers of w will give the other nth roots of unity and the higher powers will be repetitions. In particular, $w^n=1$, and hence also $w^{kn}=(w^n)^k=1^k=1$, k being any positive or negative integer. Using these relations, let us now transform the given determinant into an equivalent form by adding to the elements of the first column, the sum of

w^1 times the elements of the second column,
w^{21} times the elements of the third column,

w^{3i} times the elements of the fourth column,

....,

$w^{(n-1)i}$ times the elements of the last column.

We then have for the elements of the first column of the transformed determinant the following:

$$a+w^ib+w^{2i}c+\ \dots\ +w^{(n-1)i}n,$$
$$n+w^ia+w^{2i}b+\ \dots\ +w^{(n-1)i}m=w^i\,(a+w^ib+w^{2i}c+\ \dots\ +w^{(n-1)i}n),$$
$$m+w^in+w^{2i}a+\ \dots\ +w^{(n-1)i}l=w^{2i}(a+w^ib+w^{2i}c+\ \dots\ +w^{(n-1)i}n),$$
$$\dots\dots\dots\dots = \dots\dots\dots\dots$$
$$b+w^ic+w^{2i}d+\ \dots\ +w^{(n-1)i}a=w^{(n-1)i}(a+w^ib+w^{2i}c\ \dots\ +w^{(n-1)i}n).$$

It thus appears that $a+w^ib+w^{2i}c+\dots+w^{(n-1)i}n$ is a factor of the original determinant, and since i may have any value from 0 to $^{n-1}$ inclusive, it appears that the original determinant has n factors no two of which are equal since the successive powers of w are all different. Finally there can be no other factors since the order of the determinant is n, and the coefficient of a^n in the expanded form of the determinant is the same as the coefficient of a^n in the expanded product of its n factors. We have shown, therefore, that

$$\begin{vmatrix} a, b, c, \dots, m, n \\ n, a, b, \dots, l, m \\ m, n, a, \dots, k, l \\ b, c, d, \dots, n, a \end{vmatrix} = \prod_{i=0}^{n-1}(a+w^ib+w^{2i}c+\dots+w^{(n-1)i}n).$$

Special Cases

1. When $n=2$, we have $a^2—b^2=(a+b)(a—b)$.
2. When $n=3$, $a^3+b^3+c^3—3abc=(a+b+c)\ (a+wb+w^2c)\ (a+w^2b+wc)$, $w^3=1$.
3. When $n=4$, we have

$$a^4—b^4+c^4—d^4—2(a^2c^2—b^2d^2)—4(a^2bd—ab^2c+bc^2d—acd^2)$$
$$=(a+b+c+d)\,(a+bi—c—di)\,(a—b+c—d)\,(a—bi—c+di),$$
where $i^2=—1$.
$$=[(a+c)^2—(b+d)^2]\,[(a—c)^2+(b—d^2].$$

It should be observed that by rearranging the rows or columns of the original determinant it may be expressed as a symmetric determinant, thus

$$\begin{vmatrix} a,b,d, \dots, n \\ n,a,b, \dots, m \\ m,n,a, \dots, k \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ b,c,d, \dots, a \end{vmatrix} = (-1)^{n(n-1)/2} \begin{vmatrix} n,m,l, \dots, a \\ m,l,k, \dots, n \\ l,k,j, \dots, m \\ \\ a,n,m, \dots, b \end{vmatrix}$$

[In the "Note on Factorization", as published in the Mathematics News Letter of April, 1929, our main interest was in the fact of the existence of two different sets of prime factors (one in the field of all rational integers, i. e., R(1), the other in the field of R(w)) of every integer which can be

expressed in the form $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, or $a^3+b^3+c^3-3abc$.

Though we did not in the Note do so, we might have said that in the field R(1), factorization of such a number is unique while in R(w) it is double, that it, is decomposable into two distinct sets of prime factors in that field. To recall the example used: If $a=2, b=4, c=9$, $a^3+b^3+c^3-3abc=585$. Thus in R(1), we have

$585=3\cdot3\cdot5\cdot13$, which is the unique factored form of 585 in field R(1). But in R (w), we have not only,

$585=3\cdot3\cdot5\cdot13$

but also, $585=3\cdot5\cdot(2+4w+9w^2)(2+4w^2+9w)$. As corollary to this we have, in field R(1), $39=3\cdot13$,

while, in field R(w), $39=(2+4w+9w^2)(2+4w^2+9w)=3\cdot13$

Furthermore, it could have been pointed out that, as long as w is defined by the relation $w^2+w+1=0$, there are two and **only** two distinct sets of prime factors of the integer $a^3+b^3+c^3-3abc$, in which a, b, c, are also integers, in field R(w). The invariancy of this double factorization property for every integer capable of being expressed in the form of a symmetric determinant of order n, n any positive integer, is here elegantly shown by Professor Moritz. In precise correspondence to this is a further implication from his proof, namely: Every integer which is capable of being thrown into the form of a symmetric determinant of order n, with integral elements, has a unique double set of prime factors in the field R($\sqrt[n]{1}$), provided $\sqrt[n]{1}$ is any primitive nth root of unity.] —*S. T. S.*

ATTENTION, MATHEMATICS TEACHERS, COLLEGE, HIGH SCHOOL, GRADE, OF MISSISSIPPI AND LOUISIANA!

The joint annual meeting of the Louisiana-Mississippi Section of M. A. of A. and the Mississippi-Louisiana Branch of the National Council of Teachers of Mathematics is dated for March 7, 8—a little more than 40 days from the present date—at Cleveland, Mississippi, with the Mississippi Delta State Teachers' College booked to do the part of host. Every effort is being put forth by Chairman J. A. Hardin, of Centenary College, and Secretary-treasurer, Miss Julia Dale, Head of the Mathematics Department at the Delta State College, to make the meeting the most successful that has been held since the consolidation of these two bi-state mathematical organizations at Shreveport several years ago.

According to concerted plans of Chairman Hardin, Secretary Dale and the Mathematics News Letter Editorial Management, 400 extra copies of the present issue of the Letter are to be published and distributed, 200 among the non-subscribing mathematical people in Louisiana, and 200 among the non-subscribers in Mississippi, the distribution to be made largely in the northern sections of the States.

Due to financial handicaps of the News Letter, it is probable that the present issue—which goes out as the December number—will be the only one that will be published prior to the Cleveland meeting. This adds to the importance of the following announcements regarding the Cleveland program and arrangements for the accommodation

of the visiting delegates to the meeting, to be held Friday, March 7, 8.

Dean Hardin writes as follows: The speaker for the Friday evening program is Professor W. D. A. Westfall of the University of Missouri, Columbia, Missouri. Among the Saturday morning participants in the Section program will be Professor W. W. Elliott, of Duke University, North Carolina, Professor H. E. Buchanan, of Tulane, Major Jas. P. Cole, of Louisiana Polytechnic Institute, Professor W. Paul Webber, of L. S. U., Professor C. G. Killen, of Louisiana Normal College. As to the Louisiana-Mississippi Council Branch program, which is dated for Friday afternoon, March 7, and which is the high school portion of the joint program, no report of plans in progress has, as yet, been made.

From Secretary Dale comes the following communication from which we draw the conclusion that Miss Dale is in charge of the work of constructing the Mississippi end of the two-State mathematical program: Names of the Mississippi participants in the two-day session of Louisiana and Mississippi mathematicians, so far as concerns the Section of M. A. of A., will be announced to the public at a later date. Will you announce in the News Letter that every delegate will be cared for in the College dormitory at $1.00 for room fee? The dormitory is in suites, with bath to every four people. Meals can be secured in the college hall for 25c per person. The Delta College is giving the banquet Friday night in honor of the occasion. Would be glad for those who are planning to be present to make reservations for rooms and a plate at the banquet. The banquet will be free to all delegates. Those planning to be present should communicate as early as possible with Miss Julia Dale, Cleveland, Mississippi.

STRENGTHEN THE SECTION-COUNCIL BOND!

After the announcements from Chairman Hardin and Secretary Dale had been set up in type (see another page of the Letter) information reached us that the Louisiana-Mississippi Branch of the Council, under the direction of its officers, Mrs. B. A. Summer, Miss Norma Touchstone, Professor B. A. Tucker, have made up an excellent program for the coming meeting at Cleveland. Though we have had no news regarding its personelle, the names of these leaders should be sufficient guaranty that the Friday afternoon part of the two-day program at the Mississippi Delta College will measure up to high standards.

It is proper that we should make now the appeal we have made each year since the establishment of this close affiliation between our Louisiana-Mississippi college teachers and Louisiana-Mississippi secondary and grade teachers. The appeal is the simple one: **Let the bond between the two great classes of mathematical workers grower stronger and stronger.**

We wish that we had secured the permission of Chairman Hardin to quote the entire letter recently received by him from that nationally known figure in the mathematical world, namely Professor H. E. Slaught. His letter was written about our Section-Council affiliation, and in it he pointed out that our scheme of organized correlation of the two classes of workers, centralized as it is about the News Letter as a common organ, is being so favorably regarded by other parts of the country, that some of them have even adopted plans similar to ours as a means of stimulating mathematical interest and activity.

It is profoundly to be hoped that all college teachers who shall go to Cleveland in March will so time their arrival that they will be able to get everything which the Council Branch will have to offer on Friday afternoon. Then, during the college program of Saturday why should there not be a strong and abundant Council element "sitting in"?

When that time comes in which the secondary and the college teachers shall regularly sit together and study their mutual problems, then will have begun a new era in the history of mathematics.

—*S. T. S.*

ON THE USE OF COMPLEX NUMBERS

By W. PAUL WEBBER
Louisiana State University

By definition two complex numbers $a+ib$ and $x+iy$ ($i=\sqrt{-1}$) are equal if and only if $a=x$ and $b=y$. The equation $x+iy=a+ib$ is equivalent to two equations in real numbers, viz. $x=a$, $y=b$, x,y, a,b being any real quantities whatever. This principle is always available when an additional equation is needed to solve a problem. It is of use in various fields.

In an elementary course in differential equations there were given in the text two exercises (among others) in linear differential equations with constant coefficients, viz.,

$$d^2y/dx^2+y=\sin x$$

and

$$d^2y/dx^2+y=\cos x.$$

There is not a hint in the text (a current elementary one) that any difficulty might arise or any suggestion as to how to proceed. The result was that the whole class (not an unusually strong one) went up in the air. The reason was that the integrations required in the regular procedure could not be performed by the formulas in the table of integrals because of a zero in the denominator of a coefficient in the integral for these particular cases.

To obtain the solutions one has recourse to Euler's formulas,

$$\sin x=(e^{ix}-e^{-ix})/2i$$

and

$$\cos x=(e^{ix}+e^{-ix})2$$

By use of these the equations can be solved but the work is a little tedious for beginners. If nowever use is made of the principle stated at the beginning of this paper the solutions of both differential equations are obtainable at once and quite easily. As a result of Euler's equations or by direct expansion in series it can be shown that

$$e^{ix}=\cos x+i\sin x$$

and

$$e^{ix}=\cos x-i\sin x$$

Now writing the differential equations as

$$d^2y_1/dx^2+y_1=\sin x, \quad d^2y_2/dx^2+y'_2=\cos x,$$

we then write

$$d^2y_1/dx^2+d^2y_2/dx^2=d^2Y/dx^2, \quad \text{where } y_1+iy_2=Y.$$

Then $\quad d^2Y/dx^2+Y=i \sin x+\cos x=e^{ix}$

Now write in the regular way

$$(D+i)(D-i)Y=e^{ix}$$

Let $\quad (D-i)Y=w$

Then $\quad (D+i)w=e^{ix}$

Whence $\quad w=c_1 e^{ix}+1/2i\, e^{ix}$

Substituting this value of w in the other equation

$$(D-i)Y=c_1 e^{ix}+1/2i\, e^{ix}$$

From which by the new method gives

$$y_1+iy_2=Y=c_2e^{ix}-c_1/2i\, e^{ix}+1/2i\, xe^{ix}$$

Now replacing e^{ix} by $\cos x+i \sin x$

$$y_1+iy_2=c_2 \cos x+ic_2 \sin x+ic_1/2 \cos x+c_1/2 \sin x$$
$$-i/2\, x \cos x+x/2 \sin x.$$

Using the principle of complex numbers state above

$$y_1=c_2 \cos x+c_1/2 \sin x+x/2 \sin x$$

or $\quad y_2=c_2 \cos x+c_1 \sin x+x/2 \sin x$

since c_1 is equally arbitrary as $c_1/2$

and $\quad y_2=c_2\sin x+c_1\cos x-x/2 \cos x.$

Thus it is easier to solve both equations together by this method than to solve either one by direct use of Euler's formulas.

ON THE NOTIONS OF VELOCITY AND OF ACCELERATION

By H. L. SMITH
Louisiana State University

In the present note it is shown how the notions of vector velocity and vector acceleration may be introduced by means of the simpler notions of scalar velocity and scalar acceleration in a definite direction.

1. **Definition of Scalar Velocity and of Scalar Acceleration in a Definite Direction.** Let P_0 be a particular position of a particle P and let Q be a point distinct from P_0. On the line P_0Q set up a scale in which P_0 corresponds to 0

and Q corresponds to the distance from Q to P_0. Let s be the number in this scale which corresponds to M, the projection on P_0Q of the moving particle P. Then $(DS)_0$, $(D^2S)_0$ are defined to be the scalar velocity and the scalar acceleration of P at the position P_0 in the direction P_0Q. Here D denotes differentiation with respect to the time t and the subscript zero denotes that the derivatives are to be taken for the value of t (say t_0) for which P assumes the position P_0.

2. **Formulas for Velocity and for Scalar Acceleration.** Let us introduce a coordinate system O-X Y Z. Let the coordinates of P, P_0, Q be (xyz), $(x_0y_0z_0)$, $(x_0+a,\ y_0+b,\ z_0+c)$, respectively and let us assume that Q is at a unit disance from P_0 so that

(1) $\quad a^2+b^2+c^2=1.$

Then the coordinates of M are

$$(x_0+as,\ y_0+bs,\ z_0+cs).$$

Since MP is perpendicular to P_0 M, we have

$$a(as+x_0-x)+b(bs+y_0-y)+c(cs+z_0-z)=0,$$

from which, by aid of (1),

(2) $\quad s=a(x-x_0)+b(y-y_0)+c(z-z_0),$

Hence

(3) $\quad Ds_0=a(Dx)_0+b(Dy)_0+c\,(Dz)_0,$

(4) $\quad (D^2s)_0=a(D^2x)_0+b(D^2y)_0+c(D^2z)_0.$

Now set

(5) $\quad v_0=\sqrt{(Dx)_0^2+(Dy)_0^2+(Dz)_0^2}$

(6) $\quad j_0=\sqrt{(D^2x)_0^2+(D^2y)_0^2+(D^2z)_0^2}$

and let V and J be the points $(x_0+(Dx)_0, y_0+(Dy)_0, z_0+(Dz)_0)$, $(x_0+(D^2x)_0,\ y_0+(D^2y)_0,\ z_p+(D^2z)_0)$, respectively. Then (3), (4) may written

(7) $\quad (Ds)_0=v_0\cos\ Q\,P_0V,$

(8) $\quad (D^2s)_0=j_0\cos\ Q\,P_0J,$

respectively.

3. **The Velocity Vector.** By a formula from the calculus, v_0 is the rate of change of the distance traversed by the particle with respect to time; we call it the **velocity in the path.** By another theorem P_0V is the progressive (i. e., in direction of motion) tangent to the path. Hence

The maximum scalar velocity of P at the position P_0 is in the direction of the progressive tangent to the path of motion

and is equal to the velocity in the path at that point.

Thus the vector P_0V represents the velocity in the sence that it has the length and direction of the maximum scalar velocity.

4. **The Centre and Radius of Curvature.** In the calculus it is shown that the radius of curvature of the path of P at the position P_0 is

$$(9) \quad r_0 = \frac{v_0^2}{X_0^2 + Y_0^2 + Z_0^2}$$

where

$$\begin{aligned} X_0 &= (Dy)_0(D^2z)_0 - (Dz)_0(D^2y)_0, \\ (10) \quad Y_0 &= (Dz)_0(D^2x)_0 - (Dx)_0(D^2z)_0, \\ Z_0 &= (Dx)_0(D^2y)_0 - (Dy)_0(D^2x)_0. \end{aligned}$$

Also the centre of curvature C is the point

$$C(x_0 + r_0 f,\ y_0 + r_0 g,\ z_0 + r_0 h)$$

where

$$\begin{aligned} Hf &= (Dz)_0 Y_0 - (Dy)_0 Z_0, \\ Hg &= (Dx)_0 Z_0 - (Dz)_0 X_0, \\ (11) \quad & \\ Hh &= (Dy)_0 X_0 - (Dx)_0 Y_0, \\ H &= v_0 \sqrt{X_0^2 + Y_0^2 + Z_0^2}. \end{aligned}$$

We note that

$$(12) \quad f^2 + g^2 + h^2 = 1.$$

5. **The Acceleration Vector.** If in (4) we put

$$a = f, \quad b = g, \quad c = h$$

we get by aid of (9), (10), (11),

$$(D^2s)_0 = v_0^2/r_0.$$

Hence **the scalar acceleration in the direction of the centre of curvature is** v_0^2/r_0. It can be represented in magnitude and direction by the vector P_0N where N is the point

$$(x_0 + v_0^2/r_0 f,\ y_0 + v_0/r_0 g,\ z_0 + v_0^2/r_0 h)$$

If in (4) we put $a = l$, $b = m$, $c = n$ where

$$l = (Dx)_0/v_0, \quad m = (Dy)_0/v_0, \quad n = (Dz)_0/v_0,$$

we get

$$(D^2s)_0 = [(Dx)_0(D^2x)_0 + (Dy)_0(D^2y)_0 + (Dz)_0(D^2z)_0]/v_0 = (Dv)_0$$

where

$$v = \sqrt{(Dx)^2 + (Dy)^2 + (Dz)^2}.$$

Hence the acceleration in the direction of the progressive tangent

is $(Dv)_0$. It may be represented in magnitude and direction by the vector P T, where T is the point

$$(x_0+l(Dv)_0,\ y_0+m(Dv)_0,\ z_0+n(Dv)_0)$$

It is easily shown that the points P T J N form a rectangle of which P J is a diagonal. Hence

$$j=\overline{P\ J}=\sqrt{\overline{P_0T}^2+\overline{P_0N}^2}=\sqrt{(Dv)_0{}^2+(v_0{}^2/r_0)^2}.$$

But by (8) the vector P J represents the maximum acceleration in magnitude and direction and since it is the vector sum of the vectors P_0T, P_0N we have the following result.

The maximum scalar acceleration of P at the position P_0 is represented in magnitude and direction by the vector which is the vector sum of (1) that vector drawn from P_0 in the direction of the centre of curvature with length v^2/r_0 and (2) the vector of length $(Dv)_0$ drawn from P_0 in the direction of the progressive tangent or in the direction of the regressive tangent according as $(Dv)_0$ is positive or is negative.

COMMENT ON RECENT BOOKS

By DORA M. FORNO

1. An Arithmetic for Teachers. By William F. Roantree and Mary S. Taylor.

Publishers: The Macmillan Company, New York, 1927.

The writers of this book have produced a masterpiece. It is not surpassed by any other of its kind. They have carried out their aim, namely, that "a teacher should know something of the historical development of mathematics, since this acquaints him with the stages of mathematical progress. He should understand something of the logical development of the subject, for this leads to an appreciation of the value of sequence, to the recognition of various types of exercises, and to an understanding of the methodology of subject matter. He should understand something of the utility of the subject—'the human significance of mathematics'—since this gives him a zeal for the subject, an assurance in the value of what he presents, and a guide for the formation of good habits in the use of mathematical tools."

2. Supervised Study in Mathematics and Science. By S. Clayton Summer.

Publishers: The Macmillan Company, New York, 1922.

The author has not attempted to cover even approximately the subjects of mathematics and science, but has given most valuable suggestive lessons. He has given one or two typical outlines of a topic or subject with intimation of how the teacher may apply them further. It is a book of explicit and direct value to teachers.

3. A Laboratory Geometry. By William A. Austin.

Publishers: Scott, Foresman and Company, Chicago, 1926.

The organization of subject matter in this book seeks to achieve the major aim through a technique of which the following are the outstanding features:

(a) The consistent use of a laboratory plan for studying the subject—a plan of learning by doing.

It is a course correlating geometrical drawing and geometry.

SOLUTION OF PROBLEM

Proposed by W. PAUL WEBBER

Problem: A ladder 20 ft. long stands vertically against a wall. A cat starts up the ladder and at the same time the foot of the ladder is pulled away from the wall at the same rate that the cat climbs. What will be the greatest elevation from the ground the cat can attain while the ladder is brought to a horizontal position on the ground?

Solution

Solved by I. Maizlisn. Referring to a figure which the reader may construct if he desires, let AB be the position of the ladder t seconds after the cat begins to climb. At this instant let P be the position of the cat. IT WILL BE ASSUMED THAT THE SPEED WITH WHICH THE CAT CLIMBS THE LADDER (AS WELL AS THE SPEED OF THE FOOT OF THE LADDER) IS CONSTANT, NO MATTER WHAT THE INCLINATION OF THE LADDER, TO THE HORIZONTAL MAY BE. With this understanding, let

Z = distance OA, vertical along wall.

L = length of the ladder,

v = speed of the cat, and also the speed with which the foot of the ladder is pulled out horizontally,

X = (horizontal) distance of the foot of the ladder from the wall at any time ,and

Then, (1) $X = vt$

and (2) $Y = (v \sin u)t$

From Figure 1 it is readily seen that

$$(3) \quad \sin u = Z/L, \quad Z = \sqrt{L^2 - X^2}$$

Substituting the values of sin u and t obtained from eqs. (1) and (3) in equation (2), we obtain

$$(4) \qquad Y = (X/L)\sqrt{L^2 - X^2}$$

To find the maximum value of Y, differentiate equation (4), place the result equal to zero, and solve for X. The value of X thus obtained is to be substituted in (4). On carrying out the work we find that

$$Y = L/2$$

In the above problem, $L = 20$ ft. Hence, the maximum elevation will be

$$Y = (20\text{ft})/2 = 10 \text{ ft.}$$

Equation (4) is the equation of the path traversed by the cat.

Comment by Solver

In the above problem the character of the motions of the foot of the ladder and of the cat was the same. In a modified problem I may call attention to the fact that if the character of these two motions were **not** the same as that in the original problem, the result would not be the same.

Let the ladder occupy any position at any time t, and let P be the position of the cat at this instant. Then, S being cat's distance from foot of ladder,

$$y = S/L\sqrt{L^2 - x^2} \qquad (1)$$

As a special case let us assume that the foot of the ladder moves N times as fast as the cat. Then, $x = Ns$. Equation (1) will then become

$$y = x/NL\sqrt{L^2 - x^2} \qquad (2)$$

This will be a maximum when $x = L/\sqrt{2}$. Thus, the maximum height h in this case will be

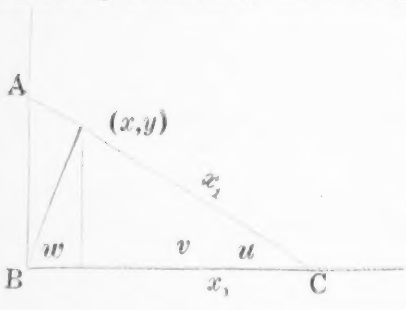
$$h \text{ max.} = 1/N \cdot L/2.$$

Passing to a more complete generalization of this modified problem we may state it as follows:

A ladder stands vertically against a wall. A cat starts up the ladder with in initial velocity U, and at the same time the foot of the ladder is pulled away from the wall with an initial velocity W. (U not equal W). If the two motions are allowed to be as general as possible, find the maximum elevation from the ground the cat will attain while the ladder is brought to a horizontal position on the ground.

A Second Solution

[Due to the interesting use made of polar coordinates the following solution of the same problem is also given.]



Solved by Frances Bailey, Louisville, Kentucky. In the figure let AC be the ladder whose foot is being pulled away from the wall BA. The point (x,y) is the cat's position at any time. Let x_1 be the distance the ladder has been pulled away from the wall, and also the distance the cat has gone up the ladder. AC=20 ft. Let u be the angle which the ladder makes with the ground.

From the figure it is clear that

$$x = x_1 - V, \quad x_1 = 20 \cos u, \quad V = x_1 \cos u,$$

and (1) $x = 20 \cos u - 20 \cos^2 u$,

(2) $y = x_1 \sin u = 20 \sin u \cos u = 10 \sin 2u$.

$$dy/du = 20 \cos 2u. \quad \text{Let } dy/du = 0.$$

Then, $\cos 2u = 0$, and therefore, $u = 45^\circ + n90^\circ$. Since, in this problem, u is restricted to values between 0° and 90° we need only to consider the value of $n = 0$.

When $u = 45^\circ$ then d^2y/du^2 is negative and for this value of u we know that y has a maximum. Substituting this value of u in (2) we find this maximum value to be 10 feet.

Equations (1) and (2) are the parametric equations of the path of the cat. To reduce this system to a single equation in polar coordinates we need only substitute $u=180-2w$ in (1) and (2), square these equations and add, and we may reduce this equation to

$$r=\pm 40 \cos w \cos 2w.$$

Solved also by F. A. Rickey, and S. T. Sanders, Jr.

SOLUTION OF PROBLEM

Proposed by W P. WEBBER

If squares are constructed on the four sides of a parallelogram exterior to it and in its plane, show that the quadrilateral determined by connecting the centres of these squares is a square.

Solved by S. T. Sanders, Jr. Let given parallelogram be A B C D, and M N O P the quadrilateral determined by the centres of the squares on its sides. Let S B and R B be two concurrent sides of one of the larger and one of the smaller squares, respectively.

Clearly, $\angle^s$AMB, BNC, COD, and DPA are rt $\angle^s$, being central angles of squares. So if we can show that $\angle$AMP=$\angle$BMN, we will have PMN a rt $\angle$, with a similar situation at the other vertices of the quadrilateral.

This suggests proving the congruency of $\triangle^s$ AMP, BMN, MCO, and OOP, since we note that the 4 sides of the quadrilateral would then be equal and the theorem proved. Examining first, $\triangle^s$ AMP, and BMN, we have,

MB=MA; and BN=AP, (semi-diagonals of equal squares).
$\angle$NBR=$\angle$DAP
$\angle$MBS=$\angle$MAB (formed by diagonals and sides of equal squares)
and $\angle$RBS=$\angle$DAB (having a mutual supplement, $\angle$ABC)

Hence, $\angle$MBN=$\angle$MAP, and the $\triangle^s$MBN and MAP are congruent. Similarly, $\triangle^s$MAP and PDO are congruent as also are $\triangle^s$PDO and OCN, whence the angles of our quadrilateral are rt . $\angle s$, the sides equal, and the figure is a square.